

Technical note on the linearity and power dependence of the diffusion coefficient in W7-AS

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Abstract. Transient electron temperature measurements of a step power experiment at W7-AS are reassessed by direct comparison of the up- and downward responses of the electron temperature. The analysis shows that the response at some distance to the center behaves linear and that the model predicted responses based on a power dependent diffusion coefficient are at variance with the measured step responses.

1. Introduction

In [1, 2, 3, 4] the dependence of the electron diffusion coefficient χ_e on electron heating power P is reported. It is shown that a model with explicit power dependence of χ_e gives a much better result in reproducing the experimental data than other temperature dependencies of χ_e on T_e and ∇T_e [2]. The diffusion dependency on the power gives a non-linear heat-flux model. Hence, for two different power levels, this results in two different diffusion coefficients, i.e. two different transient responses of the temperature.

In this Letter we show that, for a stepped power experiment at W7-AS as presented in [1, 2], the response at some distance to the source does not change non-linearly with the power. This is shown by introducing a transformation which does not affect the response. Consequently, the diffusion coefficient does not change as function of the power. Moreover, closer to the source the transformation shows that the measured responses are non-linear. In addition, comparing the measured responses with the responses based on the power dependent model, shows that the model should be more non-linear to accurately describe the measurements.

2. Step response and linearity

In this section, a transformation is introduced which allows the direct comparison of time constants and responses due to a step response. In the case of a linear time invariant model this transformation does not affect the time-constant or diffusion coefficient. In other words, if the responses are the same after the transformation is applied also the diffusion coefficients are the same. The underlying idea of this transformation is based on [5].

Assume a linear time invariant model that describes the T_e evolution due to a change in the power $P(t)$

$$\dot{T}_e(t) = AT_e(t) + BP(t), \quad (1)$$

where T_e is the temperature vector containing all spatial locations, B containing the deposition profile, and A is a tridiagonal Laplacian matrix containing the diffusion coefficient. In case of a stepped experiment

$P(t)$ is given by

$$P(t) = \begin{cases} P_\alpha & t < t^+ \text{ or } t \geq t^+ + \Delta t \\ P_\beta & t^+ \leq t < t^+ + \Delta t \end{cases}. \quad (2)$$

The two power levels in P are given by P_α and P_β where t^+ is the time instant that the step has occurred and Δt the time it takes for the step in opposite direction to occur. The time constants or the difference in diffusion coefficients for the upward and downward step can be directly compared by applying three transformations. First, shifting the response in time with Δt such that the steps overlap

$$v(t) = T_e(t + \Delta t). \quad (3)$$

Then, taking the mirrored image

$$w(t) = -v(t), \quad (4)$$

and vertically shifting to height α

$$x(t) = w(t) + \alpha + \beta. \quad (5)$$

This results in a response with the same time-constant or A as the original. This is shown in the appendix where these transformations are applied to (1) resulting in

$$\dot{x}(t) = Ax(t) + BP_\beta, \quad x(t^+) = \alpha, \quad (6)$$

assuming that when the step in power occurs, T_e is in equilibrium. The matrix A of the response $x(t)$ is the same as the time constant of the T_e response in (1). The transformation steps are also shown in Fig. 1. This transformation is used to directly compare the step responses presented in [2]. The method also works for symmetric block-waves (duty cycle 50%), but careful reinterpretation is necessary when there is a deviation from the 50% duty cycle.

3. Comparison responses W7-AS

Fig. 2 is a reproduction of Fig. 6 in the original manuscript by Hartfuß et al. [2]. The figure compares temperature measurements (ECE) with model simulations using three different dependencies on the diffusion coefficient. This figure is put forward in the original manuscript as key evidence for the power dependence of the diffusion coefficient over other ∇T_e and T_e dependencies. Fig. 2 shows

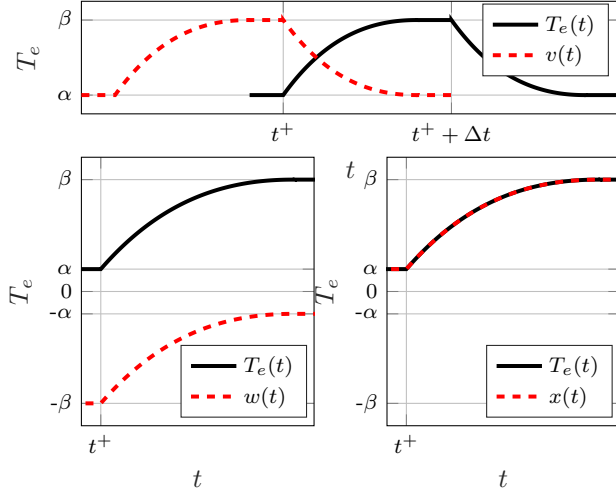


Figure 1. Transformation steps necessary to directly compare the up- and downward step responses.

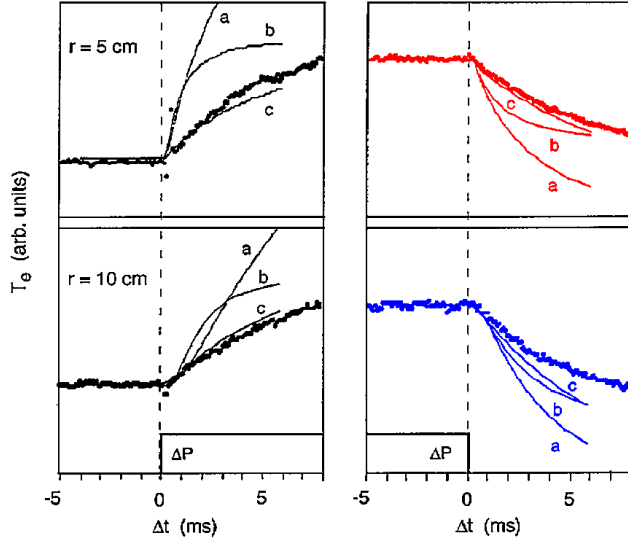


Figure 2. Reproduced with permission from [2, Fig. 6]. The original caption reads: "Time evolution of the electron temperature after switching off the heating power by ΔP . The experimental data are compared with 3 different simulations: (a): χ_e unchanged; (b): χ_e is modified proportional to $T_e^{3/2}$; and (c): the steady-state value of χ_e for the new power level is used." Note that only the color of the downward responses are changed from black to red and blue compared to the original in [2, Fig. 6].

that using the in steady-state estimated diffusion coefficients χ_e at its respective power level gives a better match with experimental data than a response in which the diffusion coefficient depends on the electron temperature $T_e^{3/2}$ or has no dependence on the temperature. The diffusion coefficient dependence on the spatial gradient of the electron temperature ∇T_e is not simulated as a strong ∇T_e dependence has been excluded in previous experiments [2]. This suggests

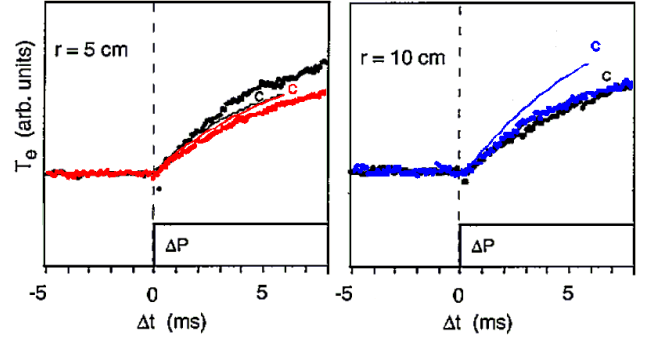


Figure 3. Adjusted with permission from [2, Fig. 6]. The difference in response to an upward step in power and a downward step in power at two different radii produced using Fig 6 in [2]. The black responses are the original responses in Fig. 2 on the left two figures. The red and blue are the transformed responses originally at the right in Fig. 2.

that the diffusion coefficient depends on power [6]. However, representing the step responses according to the method proposed in Sec. 2 will show that such a dependence cannot be justified by the stepped power experiment in W7-AS.

We apply the method in Sec. 2 to the data in Fig. 2 and the result is shown in Fig. 3. It shows the original responses for an upward step in power in black and the transformed responses for a downward step in color. The responses of T_e at $r = 5$ cm are clearly different. This also means that the time constants are different. However, comparing the c -responses, which are simulated based on a model with two different diffusion coefficients for the respective power level, shows that they are closer to an approximation of the average of both the measured responses, than describing the difference between the measured responses for an upward and downward step in the power. As such the notion that the responses can be described by a χ_e varying with the power level is questionable based on Fig. 2. On the other hand, the response at $r = 5$ cm for the higher power level is stronger than the response for a lower power level. In other words, for the same step size of the power, the change in electron temperature is more significant when the power is turned on than when it is turned off. This confirms that thermal transport has increased due to a non-linearity, which is consistent with a power dependence of thermal transport as discussed in [6, 7].

The comparison of the responses of T_e at $r = 10$ cm shows that the responses are linear with respect to the input power. This seems to suggest the same diffusion coefficient especially as the blue response (downward step in P) overlaps with the c -response (upward step in P). However, comparing all c -responses, including those at $r = 5$ cm with their corresponding measured response shows that all the c -responses are diverging from the measured responses

whereas the blue response at $r = 10$ cm is converging to the black response at $r = 10$ cm. This suggests that even the linear responses are not purely diffusive. An explanation for this last observation is already given in [2] showing that the linear measurements can be explained by taking a broader component of the ECRH power deposition profile into account. This would also be equivalent to linearizing the transport model presented in [2] (see explanation in [4]).

4. Conclusion

In this technical note, we apply a straightforward test of the linearity of step responses: by a transformation (shifting and mirroring) the responses of the electron temperature due to a step in heating power, the linearity of the system can be tested directly. The results show that close to the source the response is non-linear whereas further from the source the response behaves linear. This is consistent with perturbation theory (on-axis heating), which gives that for a small enough perturbation the response can be considered linear [8]. However, it remains unclear at this stage what the structure of the non-linearity is, as a different diffusion coefficient based on the model based c -response of $\chi_e(P)$ does not match the actual responses nor do temperature dependencies. Moreover, the linearity of the response means that if the power is changed slightly the response will scale linearly with this change. However, if the injected power is changed significantly the responses can become non-linear, which is also in line with perturbation theory.

This method can in principle also be extended to block-waves, but only if the block-wave is perfectly symmetric (duty cycle 50%) and the response no longer depends on the initial conditions, i.e., is periodic, or (as has been shown here) the response has settled. Any deviation from a symmetric block-wave will also lead to an asymmetry in the response, depending on the size of the asymmetry (difference from 50% duty cycle) still allowing to distinguish between strongly and weakly non-linear responses, but not between linear and weakly non-linear.

Based on the graph in Fig. 2 from [2] only, a full uncertainty analysis cannot be performed. Nevertheless, we are confident that such an analysis would not change the conclusion for the particular example. The reason is that as we use the same ECE-channel, i.e., same calibration and only in a local range (that of the perturbation), it is reasonable to assume that the systematic errors do not change in this temperature range. As such comparing the responses directly means these systematic errors are the same for both responses. Hence, are not relevant. The other uncertainty component are the stochastic

uncertainties, i.e., noise. These are clearly visible in Fig. 2 and are taken into account visually. If calibrated measurements are used, an uncertainty analysis of the transport as presented in [9] can be used.

Finally, note that the here presented comparison is based on the estimates of χ_e made in [2], which are based on the temperature profiles. However, as the profiles are not available to us, we base our discussions to the responses generated in [2] instead of our own estimates of χ_e .

Appendix: Preservation of time constant under applied transformations

This appendix shows the response of a linear time invariant system to a step downward can be transformed into the response to a step upward.

Consider the model as given in equation (1) in which $P(t)$ is given by (2). Furthermore, we assume that just before t^+ and just before $t^+ + \Delta t$ the temperature is at equilibrium, i.e., $T(t^+) = \alpha$ and $T(t^+ + \Delta t) = \beta$, see also Fig. 1. Hence,

$$0 = A\alpha + BP_\alpha, \quad 0 = A\beta + BP_\beta. \quad (7)$$

On the time interval $[t^+, t^+ + \Delta t)$, the temperature satisfies

$$\dot{T}_e(t) = AT_e(t) + BP_\beta, \quad T_e(t^+) = \alpha. \quad (8)$$

Assume a time shift which shifts the response as follows $v(t) = T_e(t + \Delta t)$ such that (1) becomes for $t \geq t^+$

$$\dot{v}(t) = Av(t) + BP(t + \Delta t), \quad v(t^+) = \beta. \quad (9)$$

Then mirroring the function using $w(t) = -v(t)$ gives

$$\dot{w}(t) = Aw(t) - BP(t + \Delta t), \quad w(t^+) = -\beta \quad (10)$$

and shifting to the height of the opposite T_e response by $x(t) = w(t) + (\alpha + \beta)$ gives

$$\dot{x}(t) = Ax(t) - (A(\alpha + \beta) + BP(t + \Delta t)), \quad (11)$$

with initial condition $x(t^+) = \alpha$. For $t > t^+$, we have that $P(t + \Delta t)$ equals α , and using (7), we see that

$$\begin{aligned} \dot{x}(t) &= Ax(t) - (A(\alpha + \beta) + BP(t + \Delta t)) \\ &= Ax(t) - (A(\alpha + \beta) + BP_\alpha) \\ &= Ax(t) + BP_\beta, \quad x(t^+) = \alpha, \end{aligned} \quad (6)$$

which equals (8). Hence the time-constant A of the response $x(t)$ is the same as the time-constant of T_e .

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